

Propagation and Reflection of Plane Waves in a Rotating Magneto-Elastic Fibre-Reinforced Semi Space with Surface Stress

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The present paper investigates the propagation of quasi longitudinal (qLD) and quasi transverse (qTD) waves in a magneto elastic fibre-reinforced rotating semi-infinite medium. Reflections of waves from the flat boundary with surface stress have been studied in details. The governing equations have been used to obtain the polynomial characteristic equation from which qLD and qTD wave velocities are found. It is observed that both the wave velocities depend upon the incident angle. After imposing the appropriate boundary conditions including surface stress the resultant amplitude ratios for the total displacements have been obtained. Numerically simulated results have been depicted graphically by displaying two and three dimensional graphs to highlight the influence of magnetic field, rotation, surface stress and fibre-reinforcing nature of the material medium on the propagation and reflection of plane waves.

Keywords: fibre-reinforcement, reflection, quasi longitudinal wave, quasi transverse wave, surface stress, resultant amplitude ratio.

1. Introduction

Recently, fibre-reinforcing material plays a vital role in many engineering applications due to the fact that a fibre-reinforced composite material is superior to other structural materials in many ways where high strength and stiffness in lightweight components are required. The characteristic property of a reinforced composite material is that its components act together as a single anisotropic unit as long as they remain in elastic condition. This means that the components in the reinforced composite materials are bound together so that there is no relative displacement between them. During the earthquake and similar disturbances a structure is excited

and vibration occurs with resulting oscillatory stresses. Thus the vibration and propagation of waves in a fibre-reinforced medium depend on the physical properties of the structures. It is well known in the literature that the medium of the earth exhibits various types of elastic properties. The earth may contain fibre-reinforced or self-reinforced material like hard and soft rock. The seismic signals have to travel through various materials present in the earth in the form of layers. Thus analysis of stress and strain in fibre-reinforced composite materials is an important area of research in solid mechanics.

The mechanical behaviour of many fibre-reinforced composite materials is adequately modelled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites the fibres are usually arranged in parallel straight lines. Hasin and Rosen [1] used variational method to derive bounds and expressions for the effective elastic moduli of materials reinforced by parallel, hollow, circular fibers. The plane strain or generalized plane stress deformations of a linear elastic material reinforced by a single family of parallel inextensible fibres are deduced by England *et al.* [2]. Chattopadhyay *et al.* [3] discussed the dispersion equation for horizontally polarized shear waves in a non-homogeneous self-reinforced layer over a semi-infinite self-reinforced medium with an irregularity. The problem of reflection of plane waves due to an incident longitudinal wave at a plane free fibre-reinforced thermoelastic half-space has been investigated by Singh and Zorammuana [4]. Sahu *et al.* [5] studied the propagation of horizontally polarized shear waves in a heterogeneous fiber-reinforced medium over a heterogeneous half-space under gravity. The normal mode analysis has been applied by Ailawalia *et al.* [6] to study the two dimensional deformation of fibre reinforced micropolar thermoelastic medium.

Interactions between magnetic field and strain field take place by means of the Lorentz forces which appear in the equations of motion as well as by the velocity of a material particle, moving in a magnetic field. The problems of waves and vibrations in magneto elasticity are very important due to their applications in earthquake sciences and seismology. Moreover, the earth is subject to its own magnetic field and the propagation of seismic waves on or near the surface of the earth is affected by the presence of such magnetic field. Two-dimensional thermoelastic problem of an infinite magneto-microstretch homogeneous isotropic plate is investigated by Xiong and Tian [7]. Kakar and Kakar [8] discussed Torsional waves in prestressed fibre-reinforced medium subjected to magnetic field. Said and Othman [9] applied a general model of equations of the two-temperature theory of generalized thermoelasticity to study the wave propagation in a fiber-reinforced magneto-thermoelastic medium in the context of the three-phase-lag model and Green-Naghdi theory without energy dissipation. Reflection of magneto-thermoelastic waves from a rotating elastic half-Space in generalized thermoelasticity under three theories has been studied by Othman and Song [10].

Propagation of waves in rotating media is important in many realistic problems. Effect of rotation on plane waves propagated in an elastic solid medium has its importance due to its involvements to different problems of special nature. It is known that most of the large bodies like the earth, moon and the planets have angular velocities about their polar axes. Othman and Said [11] applied Lord-Shulman (L-S) theory with one relaxation time and coupled theory to study the influence of rein-

forcement on the total deformation of a rotating thermoelastic half-space and the interaction with each other. Lofty [12] introduced the coupled theory, Lord–Schulman theory and Green–Lindsay theory to study the influences of a magnetic field and rotation on a two-dimensional problem of fibre-reinforced thermoelasticity. Effect of rotation in case of 2-D problem of the generalized thermo-viscoelasticity with two relaxation times has been investigated by Othman [13]. Bayones [14] showed the effect of rotation and initial magnetic field in a fibre-reinforced anisotropic elastic media. Othman and Abd-Elaziz [15] studied the effect of rotation and gravitational field on a micropolar magneto-thermoelastic solid in the context of the dual-phase-lag model.

It is known that the physical properties of bodies in the neighborhood of the surface are sensibly different from those of the interior. Thus the boundary surface may be regarded as a two dimensional elastic continuum which adheres to the body without slipping [16]. For example, surface tension which generally plays its role in liquid may be considered as a particular case of surface stress. Presence of surface stress on the boundary of a material medium has been detected in some particular types of crystals, where the order of magnitude agrees with the prediction made by the molecular theory. Compressive surface stress is involved in the case of shot peening of ductile metals [17]. Visible strain arises due to this process. This process is used in shaping of aircraft wing panels.

Murdoch [18] extensively investigated surface waves centred on the interface between two homogeneous linearly-elastic half-spaces which remain in non-slipping contact. Gurtin and Murdoch [19] presented a general theory of surface stress in solids. Acharya and Roy [20] showed the Effect of surface stress and irregularity of the interface on the propagation of SH-waves in the magneto-elastic crustal layer based on a solid semi space. Altenbach et. al. [21] presented a mathematical investigation of the eigenvalue problems for elastic bodies including surface stresses. Steigmann and Ogden [22] enhanced the surface elasticity theory with a flexural resistance. Considering the medium to be fibre-reinforced, problems of the propagation of plane waves and their reflection have been investigated by some authors [23–25].

For better understanding of the propagation and reflection of waves we have to consider various additional parameters due to the presence of magnetic field, rotation, fibre-reinforced material, surface stress etc. Moreover, reflection of plane waves at the flat surface with surface stress is important in estimating the correct arrival times of plane waves from the source. During our review process it is found that no reflection problem has been investigated so far considering the presence of magnetic field, rotation, fibre-reinforcement and surface stress simultaneously.

This paper is organized as follows: first, the governing equations for a linear transversely isotropic fibre-reinforced homogeneous medium are modified in the light of magnetoelastic theory and rotation of the medium. Second, the governing equations are solved analytically for two dimensional motions in $x_1 - x_3$ plane to investigate the types of speed of the propagation of plane waves. Third, using proper boundary conditions, the expressions for amplitude ratios of various reflected waves are obtained theoretically. Finally, the numerical results are illustrated graphically in two/three dimensions to highlight the effect of magnetic field, rotation, and fibre-reinforcement on the qLD and qTD wave velocities. Variations of reflected an-

gle and incident angle are presented. Two dimensional graphs are also presented to highlight the effect of fibre-reinforcement, rotation, magnetic field and surface stress on the amplitude ratios for the total displacements of reflected qLD and qTD waves when qLD or qTD waves are incident at the boundary of a semi-infinite fibre-reinforced rotating perfectly conducting magnetoelastic medium with surface stress.

2. Basic Equations

The constitutive relation for a linearly transversely isotropic fibre-reinforced elastic medium is given by Belfield *et al.* [26] as

$$\tau_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + e_{kk} a_i a_j) + 2(\mu_L - \mu_T) (a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \quad i, j, k, m = 1, 2, 3, \quad (1)$$

where τ_{ij} are the components of symmetric stress tensor, e_{ij} is the strain tensor related to displacement components u_i and is defined by $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, λ and δ_{ij} are elastic constant and Kronecker delta respectively, μ_T is shear modulus in transverse shear across the preferred direction, μ_L is the shear modulus in longitudinal shear in the preferred direction, α and β are reinforced elastic parameters and $a \equiv a_i = (a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1$.

Due to the application of initial magnetic field H , an induced magnetic field h , an induced electric field E and current density vector J are developed. The linear equations of electrodynamics for a slowly moving homogeneous electrically conducting medium may be written [27] as

$$\begin{cases} \text{curl } h = J + \varepsilon_0 \dot{E}, \\ \text{curl } E = -\mu_0 \dot{h}, \\ \text{div } h = 0, \\ E = -\mu_0 (\dot{u} \times H), \end{cases} \quad (2)$$

where ε_0, μ_0 are electric permeability and magnetic permeability,

$$u = u_i = (u_1, u_2, u_3)$$

is the displacement vector and overdot represents differentiation with respect to time. The equation of motion for the fibre-reinforced magneto elastic semi-infinite medium rotating with a uniform angular velocity $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ may be presented as

$$\tau_{ij,j} + F_i = \rho [\ddot{u} + \{\Omega \times (\Omega \times u)\} + (2\Omega \times \dot{u})]_i, \quad (3)$$

where

$$F_i = \mu_0 (J \times H)_i, \quad (4)$$

$\Omega \times (\Omega \times u)$ is the centripetal acceleration due to the time varying motion only, $(2\Omega \times \dot{u})$ is the Coriolis acceleration and ρ is the mass density of the medium.

3. Problem Formulation

We consider a semi-infinite fibre-reinforced magneto elastic rotating medium $x_3 \geq 0$ ($-\infty < x_1 < \infty, 0 < x_3 < \infty$) bounded by the plane $x_3 = 0$ which is subject to

a surface stress. The whole body is permeated by a constant magnetic field which is oriented parallel to the positive x_2 axis. The origin O is located at any point on the plane boundary surface.

The plane of symmetry of the fibre-reinforced medium is taken as x_1x_3 plane. For plain strain deformation in x_1x_3 plane the displacement components may be taken in the form $u_1 = u_1(x_1, x_3, t)$, $u_2 = 0$, $u_3 = u_3(x_1, x_3, t)$. We choose the fibre direction as $a = (1, 0, 0)$. Moreover we set $\Omega = (0, \Omega, 0)$. Hence the equations of motion are

$$\begin{aligned} \frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{13}}{\partial x_3} + \mu_0 (J \times H)_1 &= \rho [\ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3] \\ \frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{33}}{\partial x_3} + \mu_0 (J \times H)_3 &= \rho [\ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1], \end{aligned}$$

where $H = H_0 + h(x_1, x_3, t)$, $H_0 = (0, H_0, 0)$, and $h = (0, h, 0)$.

Using equations (1)–(4), the above equations of motion transform to

$$\begin{aligned} (A_{11} + \mu_0 H_0^2) \frac{\partial^2 u_1}{\partial x_1^2} + (B_2 + \mu_0 H_0^2) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + B_1 \frac{\partial^2 u_1}{\partial x_3^2} = \\ \rho \left[\left(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} \right) \frac{\partial^2 u_1}{\partial t^2} + 2\Omega \frac{\partial u_3}{\partial t} - \Omega^2 u_1 \right], \end{aligned} \quad (5)$$

$$\begin{aligned} (A_{22} + \mu_0 H_0^2) \frac{\partial^2 u_3}{\partial x_3^2} + (B_2 + \mu_0 H_0^2) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + B_1 \frac{\partial^2 u_3}{\partial x_1^2} = \\ \rho \left[\left(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} \right) \frac{\partial^2 u_3}{\partial t^2} - 2\Omega \frac{\partial u_1}{\partial t} - \Omega^2 u_3 \right], \end{aligned} \quad (6)$$

where

$$\begin{aligned} A_{11} &= \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta \\ A_{22} &= \lambda + 2\mu_T, B_1 = \mu_L \\ B_2 &= \lambda + \alpha + \mu_L \\ h &= -H_0 \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right). \end{aligned}$$

Eqs. (5) and (6) may also be represented as

$$\begin{aligned} (c_1^2 + c_A^2) \frac{\partial^2 u_1}{\partial x_1^2} + (c_2^2 + c_A^2) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + c_3^2 \frac{\partial^2 u_1}{\partial x_3^2} = \\ \left(1 + \frac{c_A^2}{c^2} \right) \frac{\partial^2 u_1}{\partial t^2} + 2\Omega \frac{\partial u_3}{\partial t} - \Omega^2 u_1, \end{aligned} \quad (7)$$

and

$$\begin{aligned} (c_4^2 + c_A^2) \frac{\partial^2 u_3}{\partial x_3^2} + (c_2^2 + c_A^2) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_3^2 \frac{\partial^2 u_3}{\partial x_1^2} = \\ \left(1 + \frac{c_A^2}{c^2} \right) \frac{\partial^2 u_3}{\partial t^2} - 2\Omega \frac{\partial u_1}{\partial t} - \Omega^2 u_3, \end{aligned} \quad (8)$$

in which: $c_1^2 = \frac{A_{11}}{\rho}$, $c_2^2 = \frac{B_2}{\rho}$, $c_3^2 = \frac{B_1}{\rho}$, $c_4^2 = \frac{A_{22}}{\rho}$, $c_A^2 = \frac{\mu_0 H_0^2}{\rho}$, $c^2 = \frac{1}{\varepsilon_0 \mu_0}$.

Introduction of the following non-dimensional quantities

$$\bar{x}_i = \frac{x_i}{c_1 \omega_1}, \bar{u}_i = \frac{u_i}{c_1 \omega_1}, \bar{h} = \frac{h}{H_0}, \bar{\tau}_{ij} = \frac{\tau_{ij}}{\mu}, \bar{t} = \frac{t}{\omega_1},$$

transforms Eqs. (7) and (8) (dropping bars for convenience) to

$$\begin{aligned} (1 + R_H) \frac{\partial^2 u_1}{\partial x_1^2} + (K + R_H) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + L \frac{\partial^2 u_1}{\partial x_3^2} = \\ \alpha_1 \frac{\partial^2 u_1}{\partial t^2} + 2R \frac{\partial u_3}{\partial t} - R^2 u_1, \end{aligned} \quad (9)$$

$$(M + R_H) \frac{\partial^2 u_3}{\partial x_3^2} + (K + R_H) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + L \frac{\partial^2 u_3}{\partial x_1^2} = \alpha_1 \frac{\partial^2 u_3}{\partial t^2} - 2R \frac{\partial u_1}{\partial t} - R^2 u_3, \quad (10)$$

where: $K = \frac{c_2^2}{c_1^2}$, $L = \frac{c_3^2}{c_1^2}$, $M = \frac{c_4^2}{c_1^2}$, $R = \Omega\omega_1$, $\alpha_1 = 1 + R_H \frac{c_1^2}{c_2^2}$, $R_H = \frac{c_A^2}{c_1^2}$.

4. Propagation of Plane Waves

For plane harmonic waves propagating in the $x_1 - x_3$ plane and incident on the boundary ($x_3 = 0$) at an angle θ with x_3 axis, the solutions of the system of Eqs. (9) and (10) may be taken as

$$\{u_1, u_3\}(x_1, x_3, t) = \{\hat{u}_1, \hat{u}_3\} e^{i\{\omega t - \xi(x_1 \sin \theta - x_3 \cos \theta)\}}, \quad (11)$$

where ξ and ω are wave number and circular frequency respectively and \hat{u}_1, \hat{u}_2 are independent of x_1, x_3, t .

Employing Eq. (11) in Eqs. (9) and (10) one obtains two linear equations in \hat{u}_1 and \hat{u}_3 in the following forms

$$\left| \begin{array}{l} (1 + R_H) \xi^2 \sin^2 \theta + L \xi^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2 \\ (K + R_H) \xi^2 \sin \theta \cos \theta - 2i\omega R \end{array} \right| \hat{u}_1 + \quad (12)$$

$$\left| \begin{array}{l} (K + R_H) \xi^2 \sin \theta \cos \theta + 2i\omega R \\ (M + R_H) \xi^2 \cos^2 \theta + L \xi^2 \sin^2 \theta - \alpha_1 \omega^2 - R^2 \end{array} \right| \hat{u}_3 = 0. \quad (13)$$

Elimination of \hat{u}_1 and \hat{u}_3 from equations (12) and (13) gives

$$\left| \begin{array}{cc} (1 + R_H) \xi^2 \sin^2 \theta + L \xi^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2 & (K + R_H) \xi^2 \sin \theta \cos \theta + 2i\omega R \\ (K + R_H) \xi^2 \sin \theta \cos \theta - 2i\omega R & (M + R_H) \xi^2 \cos^2 \theta + L \xi^2 \sin^2 \theta - \alpha_1 \omega^2 - R^2 \end{array} \right| = 0 \quad (14)$$

which may be expressed as

$$A\xi^4 + B\xi^2 + C = 0, \quad (15)$$

where

$$\begin{aligned} A &= (K + R_H)^2 \sin^2 \theta \cos^2 \theta \\ &\quad - \{(1 + R_H) \sin^2 \theta + L \cos^2 \theta\} \{(M + R_H) \cos^2 \theta + L \sin^2 \theta\}, \\ B &= (\alpha_1 \omega^2 + R^2) \{(M + R_H) \cos^2 \theta + (1 + R_H) \sin^2 \theta + L\}, \\ C &= 4\omega^2 R^2 - (\alpha_1 \omega^2 + R^2)^2. \end{aligned}$$

Equation (15) may be called polynomial characteristic equation in ξ . Solving this equation one obtains four values of ξ as $\pm\xi_1, \pm\xi_2$. Corresponding to these values of ξ , there exist two waves in descending order of their velocities namely quasi longitudinal displacement (qLD) wave with velocity v_1 and quasi transverse displacement (qTD) wave with velocity v_2 . We derive the expressions of phase velocities of these types of waves as given by $v_i = \frac{\omega}{\text{Re}(\xi_i)}$ where v_i ($i = 1, 2$) are the velocities of qLD and qTD waves respectively. Both v_1 and v_2 are functions of θ , the incident angle.

4.1. Special Cases

When fibre-reinforcement of the material medium and the rotation of the axis are both absent ($\mu_L = \mu_T, \alpha = \beta = 0, R = 0$) Eq. (15) reduces to

$$A'\xi^4 + B'\xi^2 + C' = 0,$$

where $A' = -L(1 + R_H)$, $B' = \alpha_1\omega^2(1 + L + R_H)$, $C' = -\alpha_1^2\omega^4$, $L = \frac{\mu_L}{\lambda + 2\mu_L}$, $R_H = \frac{\mu_0 H_0^2}{\lambda + 2\mu_L}$, $\alpha_1 = 1 + \frac{c_A^2}{c^2}$. In absence of fibre-reinforcement of the material medium and the magnetic field ($\mu_L = \mu_T, \alpha = \beta = 0, R_H = 0$), eq. (15) reduces to

$$A''\xi^4 + B''\xi^2 + C'' = 0,$$

where $A'' = -L$, $B'' = (\omega^2 + R^2)(1 + L)$, $C'' = -(\omega^2 - R^2)^2$, $L = \frac{\mu_L}{\lambda + 2\mu_L}$.

Again when rotation and magnetic field are absent, one may have the characteristic equation (15) as

$$A'''\xi^4 + B'''\xi^2 + C''' = 0,$$

where

$$A''' = (K^2 - L^2 - M)\sin^2\theta\cos^2\theta - LM\cos^4\theta - L\sin^4\theta,$$

$$B''' = \omega^2(M\cos^2\theta + \sin^2\theta + L)$$

and

$$C''' = -\omega^4.$$

5. Boundary Conditions

Let us consider free surface stressed boundary $x_3 = 0$ of a fibre-reinforced transversely isotropic rotating semi-infinite medium permeated by a uniform magnetic field. The boundary conditions [28] are

$$\tau_{i3} + \tau_{i3}^M + \sum_{i\alpha, \alpha} -\rho_s \frac{\partial^2 u_i}{\partial t^2} = 0, \quad (i = 1, 3) \quad \text{on } x_3 = 0, \quad (16)$$

where τ_{i3} is the conventional stress component and has the unit of force per unit area and the surface stress tensor $\sum_{i\alpha}$ has the unit of force per unit length, ρ_s is mass per unit surface area of the material boundary, τ_{i3}^M is the Maxwell's stress tensor, given by

$$\tau_{i3}^M = \mu_0 [H_i h_3 + H_3 h_i - (H_k h_k) \delta_{i3}], \quad i = 1, 3,$$

and

$$\begin{aligned} \sum_{i\alpha} &= \delta_{i\alpha} [\sigma + (\lambda_s + \sigma) u_{r,r}] + \mu_s u_{i,\alpha} + (\mu_s - \sigma) u_{\alpha,i} \quad \text{for } i, \alpha, r = 1, 2, \\ &= \sigma u_{3,\alpha} \quad \text{for } i = 3. \end{aligned}$$

We include $\bar{\rho}_s = \frac{\rho_s}{C_1 \omega_1}$ in the previous non-dimensional quantities and drop the bars to obtain two boundary conditions from eq. (16) in non-dimensional form as follows

$$N \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) + P \frac{\partial^2 u_1}{\partial x_1^2} - \frac{\partial^2 u_1}{\partial t^2} = 0, \quad (17)$$

$$(N' + R_M) \frac{\partial u_1}{\partial x_1} + (P' + R_M) \frac{\partial u_3}{\partial x_3} + R_s \frac{\partial^2 u_3}{\partial x_1^2} - \frac{\partial^2 u_3}{\partial t^2} = 0, \quad (18)$$

where $N = \frac{C_5^2}{C_1^2}$, $P = \frac{C_6^2}{C_1^2}$, $N' = \frac{C_7^2}{C_1^2}$, $P' = \frac{C_8^2}{C_1^2}$, $R_M = \frac{C_M^2}{C_1^2}$, $R_s = \frac{C_s^2}{C_1^2}$, $C_5^2 = \frac{\mu_L}{\rho_s}$, $C_6^2 = \frac{\lambda_s + 2\mu_s}{\rho_s}$, $C_7^2 = \frac{\lambda + \alpha}{\rho_s'}$, $C_8^2 = \frac{\lambda + 2\mu_T}{\rho_s}$, $C_M^2 = \frac{\mu_0 H_0^2}{\rho_s'}$, $C_s^2 = \frac{\sigma}{\rho_s}$.

6. Reflection at the Free Surface

6.1. Incident qLD Wave

When qLD wave is incident at the boundary surface one obtains reflected qLD and qTD waves. The total displacements along x_1 and x_3 axes may respectively be written in the following form

$$u_1 = A_0 \exp(iP_1) + A_1 \exp(iQ_1) + A_2 \exp(iQ_2), \quad (19)$$

$$u_3 = D_0 \exp(iP_1) + D_1 \exp(iQ_1) + D_2 \exp(iQ_2), \quad (20)$$

where the phase factors associated with the incident qLD, reflected qLD and reflected qTD waves are respectively given by

$$\begin{cases} P_1 = \omega t - \xi_1 (x_1 \sin \theta - x_3 \cos \theta), \\ Q_1 = \omega t - \xi_1 (x_1 \sin \theta + x_3 \cos \theta), \\ Q_2 = \omega t - \xi_2 (x_1 \sin \theta_1 + x_3 \cos \theta_1), \end{cases} \quad (21)$$

in which θ is the angle of incident and the angle of reflection of qLD waves where as θ_1 is the reflected angle of qTD wave. The amplitude factors in the direction of x_1 and x_3 axes associated with incident qLD and reflected qLD and reflected qTD waves are respectively written in pairs (A_0, D_0) , (A_1, D_1) , (A_2, D_2)

Using equations (19) and (20) in equations (9) and (10) one obtains the following equations corresponding to equation (12)

$$\begin{cases} \{(1 + R_H) \xi_1^2 \sin^2 \theta + L \xi_1^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2\} A_0 + \{(K + R_H) \xi_1^2 \sin \theta \cos \theta - 2i\omega R\} D_0 = 0, \\ \{(1 + R_H) \xi_1^2 \sin^2 \theta + L \xi_1^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2\} A_1 - \{(K + R_H) \xi_1^2 \sin \theta \cos \theta + 2i\omega R\} D_1 = 0, \\ \{(1 + R_H) \xi_2^2 \sin^2 \theta_1 + L \xi_2^2 \cos^2 \theta_1 - \alpha_1 \omega^2 - R^2\} A_2 - \{(K + R_H) \xi_2^2 \sin \theta_1 \cos \theta_1 + 2i\omega R\} D_2 = 0. \end{cases} \quad (22)$$

Another set of similar equation may also be obtained but due to consistency condition these equations will coincide with those in Eq. (22) from which it follows

$$A_0 = \zeta_0 D_0, A_1 = \zeta_1 D_1, A_2 = \zeta_2 D_2, \quad (23)$$

where

$$\begin{cases} \zeta_0 = \frac{\{(K + R_H) \xi_1^2 \sin \theta \cos \theta - 2i\omega R\}}{\{(1 + R_H) \xi_1^2 \sin^2 \theta + L \xi_1^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2\}}, \\ \zeta_1 = -\frac{\{(K + R_H) \xi_1^2 \sin \theta \cos \theta + 2i\omega R\}}{\{(1 + R_H) \xi_1^2 \sin^2 \theta + L \xi_1^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2\}}, \\ \zeta_2 = -\frac{\{(K + R_H) \xi_2^2 \sin \theta_1 \cos \theta_1 + 2i\omega R\}}{\{(1 + R_H) \xi_2^2 \sin^2 \theta_1 + L \xi_2^2 \cos^2 \theta_1 - \alpha_1 \omega^2 - R^2\}}. \end{cases} \quad (24)$$

Applying equations (19)-(21) in (17) and (18) one may have the following

$$\begin{aligned} & \left| (i\xi_1 N \cos \theta - \xi_1^2 P \sin^2 \theta + \omega^2) A_0 + (i\xi_1 N \sin \theta) D_0 \right| e^{iQ_1(x_1, 0)} + \\ & \left| (-i\xi_1 N \cos \theta - \xi_1^2 P \sin^2 \theta + \omega^2) A_1 + (i\xi_1 N \sin \theta) D_1 \right| e^{iQ_1(x_1, 0)} + \\ & \left| (-i\xi_2 N \cos \theta_1 - \xi_2^2 P \sin^2 \theta_1 + \omega^2) A_2 + (i\xi_2 N \sin \theta_1) D_2 \right| e^{iQ_2(x_1, 0)} = 0, \end{aligned} \quad (25)$$

and

$$\begin{aligned} & [i\xi_1 (N' + R_M) \sin \theta A_0 + \{i\xi_1 (P' + R_M) \cos \theta - \xi_1^2 R_s \sin^2 \theta + \omega^2\} D_0] e^{iQ_1(x_1, 0)} + \\ & [i\xi_1 (N' + R_M) \sin \theta A_1 + \{-i\xi_1 (P' + R_M) \cos \theta - \xi_1^2 R_s \sin^2 \theta + \omega^2\} D_1] e^{iQ_1(x_1, 0)} + \\ & [i\xi_2 (N' + R_M) \sin \theta A_2 + \{-i\xi_2 (P' + R_M) \cos \theta_1 - \xi_2^2 R_s \sin^2 \theta_1 + \omega^2\} D_2] e^{iQ_2(x_1, 0)} = 0, \end{aligned} \quad (26)$$

in which

$$P_1(x_1, 0) = Q_1(x_1, 0). \quad (27)$$

Assuming

$$Q_1(x_1, 0) = Q_2(x_1, 0), \quad (28)$$

one may obtain

$$\frac{\sin \theta}{\xi_2} = \frac{\sin \theta_1}{\xi_1}. \quad (29)$$

With the help of equations (23) and (28), equations (25) and (26) may be written as following

$$l_0 D_0 + l_1 D_1 + l_2 D_2 = 0, \quad (30)$$

$$m_0 D_0 + m_1 D_1 + m_2 D_2 = 0, \quad (31)$$

where

$$\begin{cases} l_0 = (i\xi_1 N \cos \theta - \xi_1^2 P \sin^2 \theta + \omega^2) \zeta_0 - i\xi_1 N \sin \theta, \\ l_1 = (-i\xi_1 N \cos \theta - \xi_1^2 P \sin^2 \theta + \omega^2) \zeta_1 - i\xi_1 N \sin \theta, \\ l_2 = (-i\xi_2 N \cos \theta_1 - \xi_2^2 P \sin^2 \theta_1 + \omega^2) \zeta_2 - i\xi_2 N \sin \theta_1, \\ m_0 = \{-i\xi_1 (N' + R_M) \sin \theta\} \zeta_0 + \{i\xi_1 (P' + R_M) \cos \theta - \xi_1^2 R_s \sin^2 \theta + \omega^2\}, \\ m_1 = \{-i\xi_1 (N' + R_M) \sin \theta\} \zeta_1 + \{-i\xi_1 (P' + R_M) \cos \theta - \xi_1^2 R_s \sin^2 \theta + \omega^2\}, \\ m_2 = \{-i\xi_2 (N' + R_M) \sin \theta_1\} \zeta_2 + \{-i\xi_2 (P' + R_M) \cos \theta_1 - \xi_2^2 R_s \sin^2 \theta_1 + \omega^2\}. \end{cases} \quad (32)$$

From equations (30) and (31) one may have the following

$$\frac{D_1}{D_0} = \frac{l_2 m_0 - l_0 m_2}{l_1 m_2 - l_2 m_1}, \quad \frac{D_2}{D_0} = \frac{l_0 m_1 - l_1 m_0}{l_1 m_2 - l_2 m_1}, \quad (33)$$

and hence from equation (23) we obtain

$$\frac{A_1}{A_0} = \frac{\zeta_1 (l_2 m_0 - l_0 m_2)}{\zeta_0 (l_1 m_2 - l_2 m_1)}, \quad \frac{A_2}{A_0} = \frac{\zeta_2 (l_0 m_1 - l_1 m_0)}{\zeta_0 (l_1 m_2 - l_2 m_1)}. \quad (34)$$

From equations (33) and (34) one obtains the resultant amplitude ratio for the total displacement for reflected qLD waves when qLD wave is incident at the boundary as

$$Z_1^{PP} = \left| \frac{l_2 m_0 - l_0 m_2}{l_1 m_2 - l_2 m_1} \right| \left[\frac{\zeta_1^2 + 1}{\zeta_0^2 + 1} \right]^{\frac{1}{2}}, \quad (35)$$

and the resultant amplitude ratio for the total displacement for reflected qTD waves when qLD wave is incident at the boundary as

$$Z_2^{PS} = \left| \frac{l_0 m_1 - l_1 m_0}{l_1 m_2 - l_2 m_1} \right| \left[\frac{\zeta_2^2 + 1}{\zeta_0^2 + 1} \right]^{\frac{1}{2}}. \quad (36)$$

6.2. Incident qTD Waves

We may assume the displacements as

$$\begin{cases} u_1 = A_0 \exp(iP_1) + A_1 \exp(iQ_1) + A_2 \exp(iQ_2), \\ u_3 = D_0 \exp(iP_1) + D_1 \exp(iQ_1) + D_2 \exp(iQ_2), \end{cases} \quad (37)$$

where the phase factors are

$$\begin{cases} P_1 = \omega t - \xi_2 (x_1 \sin \theta_1 - x_3 \cos \theta_1), \\ Q_1 = \omega t - \xi_1 (x_1 \sin \theta + x_3 \cos \theta), \\ Q_2 = \omega t - \xi_2 (x_1 \sin \theta_1 + x_3 \cos \theta_1), \end{cases} \quad (38)$$

in which incident and reflected qTD waves make an angle θ_1 with x_3 axis and reflected qLD waves make an angle θ with the same axis.

From equation (38) we see $P_1(x_1, 0) = Q_2(x_1, 0)$. Assuming as equation (28) we also obtain $\frac{\sin \theta}{\xi_2} = \frac{\sin \theta_1}{\xi_1}$.

Insertion of equation (37) in equation (9) leads to

$$A_0 = \zeta'_0 D_0, \quad A_1 = \zeta_1 D_1, \quad A_2 = \zeta_2 D_2, \quad (39)$$

where

$$\begin{aligned} \zeta'_0 &= \frac{\{(K + R_H) \xi_2^2 \sin \theta_1 \cos \theta_1 - 2i\omega R\}}{\{(1 + R_H) \xi_2^2 \sin^2 \theta_1 + L\xi_2^2 \cos^2 \theta_1 - \alpha_1 \omega^2 - R^2\}}, \\ \zeta_1 &= \frac{-\{(K + R_H) \xi_1^2 \sin \theta \cos \theta + 2i\omega R\}}{\{(1 + R_H) \xi_1^2 \sin^2 \theta + L\xi_1^2 \cos^2 \theta - \alpha_1 \omega^2 - R^2\}}, \\ \zeta_2 &= \frac{-\{(K + R_H) \xi_2^2 \sin \theta_1 \cos \theta_1 + 2i\omega R\}}{\{(1 + R_H) \xi_2^2 \sin^2 \theta_1 + L\xi_2^2 \cos^2 \theta_1 - \alpha_1 \omega^2 - R^2\}}. \end{aligned}$$

Applying the boundary conditions equations (17) and (18) and proceeding similarly as before we may have

$$\frac{D_1}{D_0} = \frac{l_2 m'_0 - l'_0 m_2}{l_1 m_2 - l_2 m_1}, \quad \frac{D_2}{D_0} = \frac{l'_0 m_1 - l_1 m'_0}{l_1 m_2 - l_2 m_1}, \quad (40)$$

and

$$\frac{A_1}{A_0} = \frac{\zeta_1 (l_2 m'_0 - l'_0 m_2)}{\zeta'_0 (l_1 m_2 - l_2 m_1)}, \quad \frac{A_2}{A_0} = \frac{\zeta_2 (l'_0 m_1 - l_1 m'_0)}{\zeta'_0 (l_1 m_2 - l_2 m_1)}, \quad (41)$$

where

$$\begin{aligned} l'_0 &= (i\xi_2 N \cos \theta_1 - \xi_2^2 P \sin^2 \theta_1 + \omega^2) \zeta'_0 - i\xi_2 N \sin \theta_1, \\ l_1 &= (-i\xi_1 N \cos \theta - \xi_1^2 P \sin^2 \theta + \omega^2) \zeta_1 - i\xi_1 N \sin \theta, \\ l_2 &= (-i\xi_2 N \cos \theta_1 - \xi_2^2 P \sin^2 \theta_1 + \omega^2) \zeta_2 - i\xi_2 N \sin \theta_1, \\ m'_0 &= \{-i\xi_2 (N' + R_M) \sin \theta_1\} \zeta'_0 + \{i\xi_2 (P' + R_M) \cos \theta_1 - \xi_2^2 R_s \sin^2 \theta_1 + \omega^2\}, \\ m_1 &= \{-i\xi_1 (N' + R_M) \sin \theta\} \zeta_1 + \{-i\xi_1 (P' + R_M) \cos \theta - \xi_1^2 R_s \sin^2 \theta + \omega^2\}, \\ m_2 &= \{-i\xi_2 (N' + R_M) \sin \theta_1\} \zeta_2 + \{-i\xi_2 (P' + R_M) \cos \theta_1 - \xi_2^2 R_s \sin^2 \theta_1 + \omega^2\}. \end{aligned}$$

The resultant amplitude ratios for the total displacement for reflected qLD and qTD waves when qTD wave is incident at the boundary may respectively be obtained from eqs. (40) and (41) as

$$Z_1^{SP} = \left| \frac{l_2 m'_0 - l'_0 m_2}{l_1 m_2 - l_2 m_1} \right| \left[\frac{\zeta_1^2 + 1}{\zeta_0'^2 + 1} \right]^{\frac{1}{2}}, \quad (42)$$

and

$$Z_2^{SS} = \left| \frac{l'_0 m_1 - l_1 m'_0}{l_1 m_2 - l_2 m_1} \right| \left[\frac{\zeta_2^2 + 1}{\zeta_0'^2 + 1} \right]^{\frac{1}{2}}. \quad (43)$$

7. Numerical Results and Discussion

To study the combined effect due to the presence of rotation, initial magnetic field, fibre-reinforcement and surface stress as well as their individual effects on the propagation and reflection of plane waves we now present some numerical results on the basis of the theoretical results obtained in the preceding section. We use the following physical constants [29] as

$$\begin{aligned} \lambda &= 5.65 \times 10^{10} \text{ N/m}^2, \mu_L = 5.66 \times 10^{10} \text{ N/m}^2, \mu_T = 2.46 \times 10^{10} \text{ N/m}^2, \\ \alpha &= -1.28 \times 10^{10} \text{ N/m}^2, \beta = 220.90 \times 10^{10} \text{ N/m}^2, \rho = 2.66 \times 10^3 \text{ N/m}^3, \\ \omega &= 2/\text{sec}, \lambda_S = 7 \times 10^3 \text{ N/m}, \mu_S = 8 \times 10^3 \text{ N/m}, \\ \rho_S &= 7 \times 10^{-4} \text{ kg/m}^2, \sigma = 110 \text{ N/m}. \end{aligned}$$

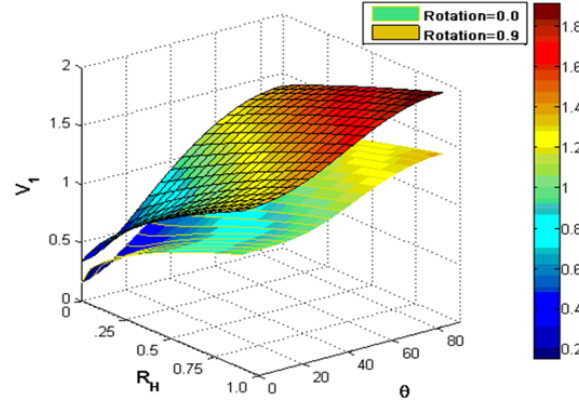


Figure 1 Variation of qLD wave velocity v_1 against R_H and θ

Numerical technique has been adopted to calculate phase velocities v_1 and v_2 of qLD and qTD waves respectively which are seen to exist in the considered model. These distributions are shown in Figs. 1-4. Plots of reflected angle θ_1 against incident angle θ are shown in Fig. 5. Variations of amplitude ratios for reflected waves against the angle of incidence have been depicted in Figs. 6-13. All the figures are self-explanatory. However, a few figures are analysed and some peculiarities are pointed out.

The three dimensional Figs. 1 and 2, each containing two surfaces, depict the variations of qLD and qTD wave velocities v_1 and v_2 respectively against θ and magnetic field parameter (R_H) for two values of rotation ratio $R = 0, 0.9$.

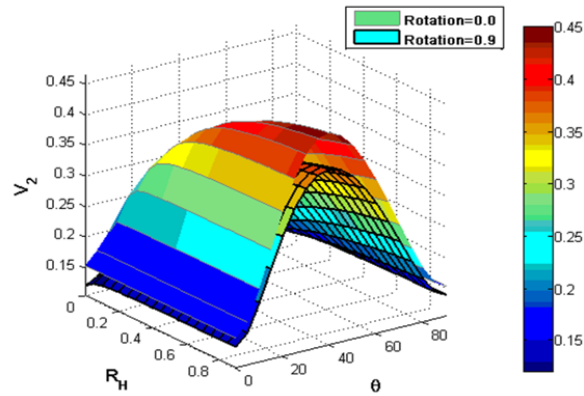


Figure 2 Variation of qTD wave velocity v_2 against R_H and θ

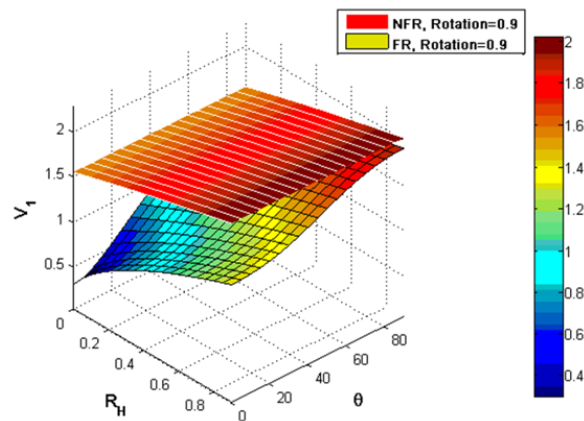


Figure 3 Variation of v_1 against R_H and θ

From the Fig.1 it is clear that rotation causes increment of v_1 in the fibre-reinforced medium for any particular value of θ and R_H . Moreover, v_1 increases monotonically with the increase of θ for any particular value of R_H and slowly increases with the increase of R_H for a fixed value of θ . From Fig. 2 it is observed that decrements of v_2 always occur due to the presence of rotation. Such diminution increases in the interval $(0^\circ < \theta < 45^\circ)$, decreases in the interval $(45^\circ < \theta < 90^\circ)$ as θ increases and attains its maximum value near $\theta = 45^\circ$. For a particular value of R_H , in presence or absence of rotation, v_2 increases sharply with θ and attains its peak value at $\theta = 45^\circ$ while for values of $\theta > 45^\circ$, v_2 decreases. For any fixed value of θ hardly any change of v_2 is noticed due to the variation of R_H .

To exhibit clearly the effect of fibre-reinforcement on wave velocities Figs. 3 and 4 have been presented. These figures depict the variations of v_1 and v_2 respectively against θ and R_H for rotation parameter $R = 0.9$. From the Fig. 3 it is very much clear that v_1 is always diminished due to the presence of fibre-reinforcement(FR)

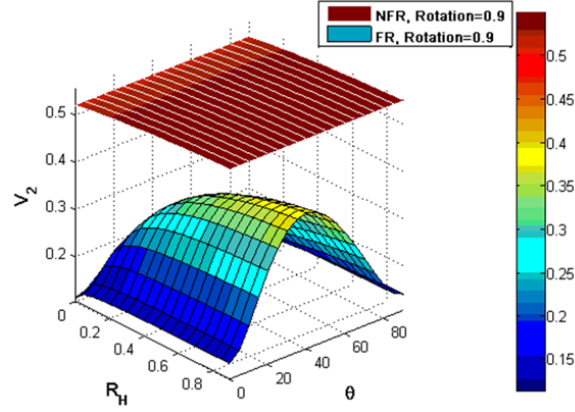


Figure 4 Variation of v_2 against R_H and θ

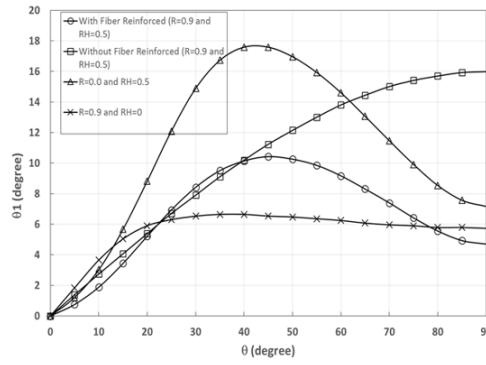


Figure 5 Variation of reflected angle θ_1 against incident angle θ

for particular values of θ , R_H and R . Such diminution diminishes with the increase of θ for any particular value of R_H . For any particular value of θ , in presence of fibre-reinforcement, v_1 slowly increases with the increase of R_H . In the absence of fibre-reinforcement(NFR), such increment is negligible. In the absence of fibre-reinforcement, Fig. 4 indicates that variations of v_2 is negligibly small due to the variations of θ and R_H . In presence of fibre-reinforcement, for any particular value of R_H , v_2 quickly increases to reach its highest value near $\theta = 45^\circ$. After reaching its highest value, decrement of v_2 occurs as θ increases.

Figure 5 illustrates the variations of θ_1 against θ in presence or absence of fibre-reinforcement, rotation and magnetic field. Comparing the curves marked by circles and squares it is noticed that the presence of fibre-reinforcement causes (i) decrements of θ_1 in $0^\circ < \theta < 23^\circ$, (ii) increments of θ_1 in $23^\circ < \theta < 40^\circ$ and (iii) decrements of θ_1 in $40^\circ < \theta < 90^\circ$ as θ increases in the respective intervals. In the absence of rotation when $R_H = .05$ and fibre-reinforcement is present, it is seen from the curve marked by triangle that θ_1 continuously increases as θ increases in $0^\circ < \theta < 45^\circ$ to reach its highest value nearly at $\theta = 45^\circ$. For $\theta > 45^\circ$, θ_1 in

this case diminishes as θ increases in $45^\circ < \theta < 90^\circ$. In the absence of magnetic field when $R = 0.9$ and fibre-reinforcement is present, θ_1 increases in the absence of magnetic field, marked by crossed line, in $0^\circ < \theta < 23^\circ$ and very small variations in θ_1 is observed when $\theta > 23^\circ$.

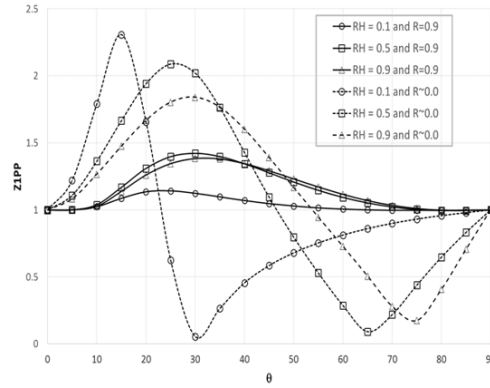


Figure 6 Variation of Z_1^{PP} against θ

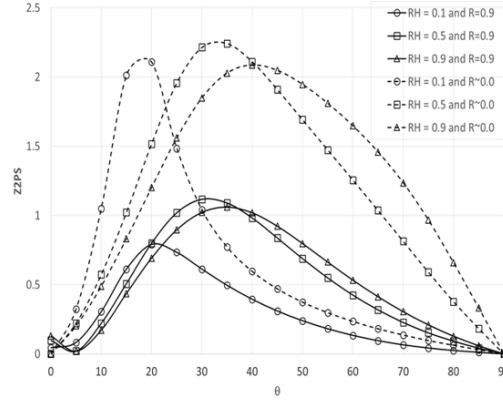


Figure 7 Variation of Z_2^{PS} against θ

Figure 6 shows the variations of Z_1^{PP} with the incident angle for incident qLD waves propagated in a fibre-reinforced semi-infinite medium with surface stress at the boundary for different values of magnetic parameter ($R_H = 0.1, 0.5, 0.9$) in presence ($R = 0.9$) or absence ($R = 0$) of rotation. Figure 7 displays the variation of Z_2^{PS} against θ under the same situation as given for Fig. 6.

Figure 8 illustrates the variation of Z_1^{PP} with θ for waves propagated in a magnetoelastic ($R_H = 0.5$) rotating ($R = 0.9$) medium under the following situations:

- (i) FR+SS indicates the presence of both fibre-reinforcement (FR) and surface stress (SS).

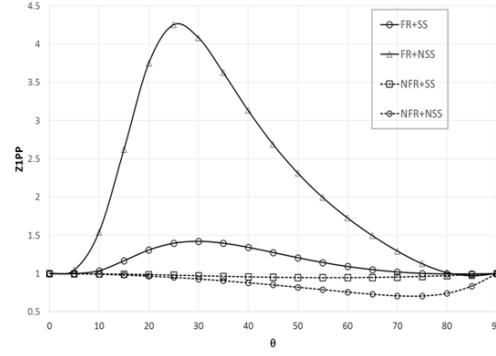


Figure 8 Variation of Z_1^{PP} against θ

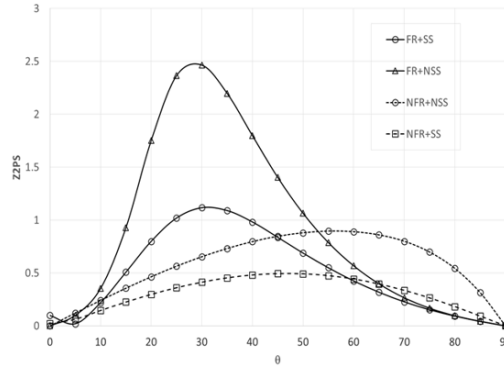
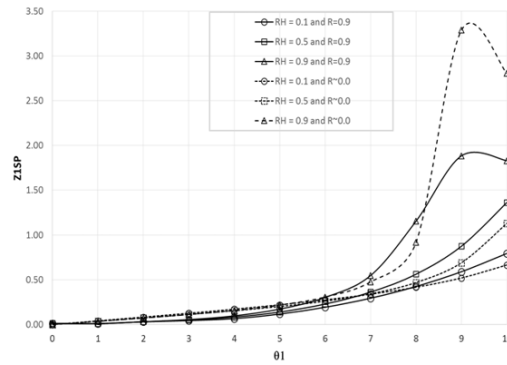
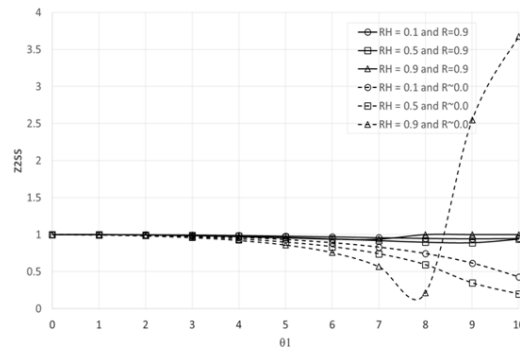
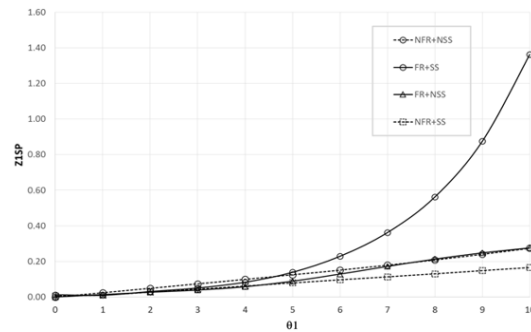


Figure 9 Variation of Z_2^{PS} against θ

- (ii) FR+NSS indicates the presence of fibre-reinforcement and absence surface stress.
- (iii) NFR+SS indicates the absence of fibre-reinforcement but presence of surface stress and
- (iv) NFR+NSS means absence of both fibre-reinforcement and surface stress.

Figure 9 is the illustration of the variations of Z_2^{PS} with θ under the same situation as expressed for Fig. 8. Figure 10 shows plots of Z_1^{SP} against θ_1 for the incident qTD wave under the same situation as written for Fig. 6 propagated in the fibre-reinforced semi-infinite medium with surface stress at the boundary for different values of magnetic parameter in absence or presence of rotation.

Figure 11 explains the variation of Z_2^{SS} with respect to θ_1 under the same consideration of magnetic parameter and rotation as chosen for Fig. 10. Figure 12 displays the variation of Z_1^{SP} against θ_1 for incident qTD wave under the same situation as in Fig. 8. Figure 13 reveals the behaviour of Z_2^{SS} with respect to θ_1 for incident qTD wave under the same situation as in Fig. 8.

Figure 10 Variation of Z_1^{SP} against θ_1 Figure 11 Variation of Z_2^{SS} against θ_1 Figure 12 Variation of Z_1^{SP} against θ_1

8. Conclusions

In the present paper the problem of propagation and reflection of plane waves in a rotating magnetoelastic fibre-reinforced semi-infinite medium in presence of surface stress has been investigated. It is observed that two types of coupled waves i.e., qLD and qTD waves can exist in this case. Both the wave velocities have been found out

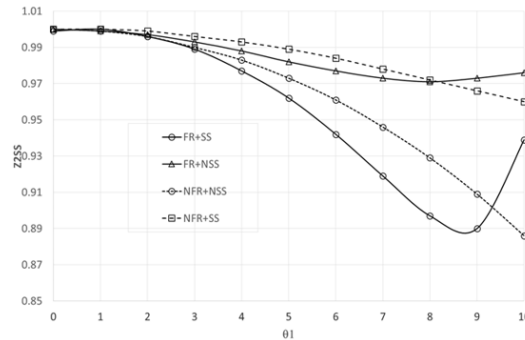


Figure 13 Variation of Z_2^{SS} against θ_1

and their variations against magnetoelastic parameter and incident angle have been studied critically, in presence or absence of rotation and fibre-reinforcement, with the help of surface plots. Variations of resultant amplitude ratios under different situations have been illustrated in details by drawing different curves in different figures. It is found that magnetic field, rotation, fibre-reinforcement and surface stress play their respective vital role in the modulation of amplitude ratios. Finally, the present paper which includes magnetic field, rotation, fibre-reinforcement and surface stress simultaneously may be considered as a more generalized one in the field of wave propagation and reflection of plane waves. Special cases have been studied with due importance. It is concluded that the presence of fibre-reinforcement, magnetic field and rotation modulates both the wave velocities to a considerable extent. Amplitude ratios are influenced by the presence of surface stress in addition. Further modulations take place due to their combined effect.

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